Efficient Estimation and Testing of Alternative Models of Currency Futures Contracts

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Abstract An efficient systems approach is used to estimate and test two alternative models regarding the pricing of Australian dollar futures contracts traded on the International Monetary Market of the Chicago Mercantile Exchange. Cointegrating relationships among the Australian dollar spot and futures prices, and the US and Australian risk-free rates of interest, suggest alternative error-correction representations for the Cost-of-Carry model which, with appropriate zero restrictions, yields the Unbiased Expectations Hypothesis. A structural break in the futures price series permits testing of appropriate models for the full sample in the presence of the break, for the full sample without explicitly modelling the break, and for two separate sub-samples created by the structural break. The restricted and unrestricted Cost-of-Carry formulations are estimated for all sample sets, and the basis of the tests of zero restrictions, the Cost-of-Carry model is found to be empirically superior to the Unbiased Expectations Hypothesis for the four sample sets considered, regardless of the number of cointegrating relations.

1. INTRODUCTION

Two well-known approaches to modelling forward and futures prices are the Cost-of-Carry model and the Unbiased Expectations Hypothesis. Although the two models may be viewed as alternative, perhaps complementary, perspectives regarding the same phenomenon, establishing evidence as to their relative merits presents different statistical properties when the time series properties of the data are ignored. The primary purpose of this paper is to examine empirically these two models for currency futures using a systems approach to both estimation and testing. Both models can be tested directly within the cointegration framework when the spot and futures prices and the relevant interest rates contain stochastic trends. There are two distinct advantages to using such a systems approach: (i) the systems approach is more efficient in estimation than is the standard single-equation approach; (ii) subject to appropriate cross-equation restrictions, the Unbiased Expectations Hypothesis is nested within the Cost-of-Carry model, and the restrictions can be tested using standard asymptotic methods.

Time series data on the Australian dollar futures and spot prices from September 1990 to July 1996 are used in the empirical analysis. This set of data is interesting because a structural break is observed in both the futures and spot prices over the full sample period, with the break corresponding to the recovery of the Australian economy in late 1993. Such a break permits estimation of appropriate futures pricing models for the full sample in the presence of the break, for the full sample without explicitly modelling the break, and for the two sub-samples created by accommodating the structural break.

Many financial market time series, such as those examined in this paper, contain stochastic trends, and are denoted as I(1) in the time series literature (a scalar time series, y_t, has a stochastic trend if its first difference, $y_t - y_{t-1}$, has a stationary invertible ARMA representation plus a deterministic component). In their seminal paper, Engle and Granger (1987) introduced the theory of cointegrated processes as a means of testing long-run theories among non-stationary variables. Subsequently, much attention in the empirical finance literature has been devoted to the possibility of two or more assets which might share the same stochastic trend, in which case the assets would be cointegrated. In the case of the futures market, when buying or selling a futures contract, a trader agrees to receive or deliver a given commodity at a certain time in the future for a price that is determined in the present. In such circumstances, it is not suprising that a long-run relationship between the futures and spot prices is expected to prevail for the underlying asset at the delivery date, as specified in the futures contract. This is the price discovery role of the futures market.

Cointegration is important because the presence of common stochastic trends restricts the set of statistical models that can be used to test economic theories. These restrictions become particularly important when testing for market efficiency using time series data, where the use of error-correction models becomes necessary. Error-correction models can be interpreted as formulations in which the current change in a non-stationary variable depends on the current and lagged changes in all variables in the system and on the deviation from the long-run equilibrium in the previous period where an equilibrium relationship exists. The difference operator is commonly applied to achieve stationarity in the data but, if the variables are cointegrated, standard methods of statistical inference are rendered inappropriate. Since this problem was not widely recognized until the 1980s, many empirical studies involving non-stationary variables seem to have drawn invalid inferences based on inappropriate asymptotic distributions.

The plan of the paper is as follows. Section 2 provides the framework for testing the alternative formulations of the Unbiased Expectations Hypothesis and the Cost-of-Carry model. The data and the various sub-samples are described in Section 3, and the empirical results are presented in Section 4. Concluding remarks are given in Section 5.

2. TESTING THE ALTERNATIVE FORMULATIONS

As mentioned above, the Unbiased Expectations Hypothesis and the Cost-of-Carry model can be estimated and tested directly even when the futures price, spot price, and the domestic and foreign risk-free rates of interest contain stochastic trends. In what follows, we shall only use one lagged difference in the error-correction representation and assume that there are one or two cointegrating relations between the variables. Extensions to higher-order vector autoregressive systems can be accommodated in a straightforward manner.

Let f_t be the (logarithmic) price of a one-period ahead futures contract at time t, s_t be the (logarithmic) spot price at time t, and r_t^d and r_t^f be the one-period domestic and foreign interest rates at time t. When s_t, f_t, r_t^d and r_t^f are I(1), and there is a cointegrating relation between s_t and f_t, the Unbiased Expectations Hypothesis is given as equations (1a)-(1b) in the following system:

$$\Delta s_t = a_0 + a_1 \Delta s_{t-1} + a_2 \Delta f_t + a_3(s_{t-1} - b_1 f_{t-1}) + \varepsilon_t^s \tag{1a}$$

$$\Delta s_t = a_0' + a_1' \Delta s_t + a_2' \Delta f_{t-1} + a_3'(s_{t-1} - b_1 f_{t-1}) + \varepsilon_t^f \tag{1b}$$

$$\Delta r_t^d = a_0'' + a_1'' \Delta r_{t-1}^d + a_2'' \Delta r_t^f + \varepsilon_t^{r^d} \tag{1c}$$

$$\Delta r_t^f = a_0''' + a_1''' \Delta r_t^d + a_2''' \Delta r_{t-1}^f + \varepsilon_t^{r^f} \tag{1d}$$

where $s_{t-1} - b_1 f_{t-1}$ is the one-period lagged error-correction term between the spot and futures prices, and a_3 and a_3' are the adjustment coefficients corresponding to $(\Delta s_t, \Delta f_t)$. No cointegrating relation is assumed between r_t^d and r_t^f, so that interest rate parity is not required in the specification of (1c)-(1d). As such, the simple time series specifications of (1c)-(1d) are arbitrary under the null hypothesis of the Unbiased Expectations Hypothesis, but serves the purpose of enabling the Unbiased Expectations

Hypothesis to be tested against the alternative Cost-of-Carry model. It is essential only that the current and lagged values of the spot and futures prices are not present in (1c)-(1d). However, the power of the test of the null hypothesis of Unbiased Expectations may be affected by the choice of the alternative hypothesis.

When s_t, f_t, r_t^d and r_t^f are all I(1), the Cost-of-Carry model for the currency futures may be formulated as follows:

$$\begin{aligned}\Delta s_t &= c_0 + c_1\Delta s_{t-1} + c_2\Delta f_t + c_3\Delta r_t^d + c_4\Delta r_t^f \\ &\quad + c_5(s_{t-1} - d_1 f_{t-1} - d_2 r_{t-1}^d - d_3 r_{t-1}^f) \\ &\quad + \varepsilon_t^s \end{aligned} \tag{2a}$$

$$\begin{aligned}\Delta f_t &= c_0' + c_1'\Delta s_t + c_2'\Delta f_{t-1} + c_3'\Delta r_t^d + c_4'\Delta r_t^f \\ &\quad + c_5'(s_{t-1} - d_1 f_{t-1} - d_2 r_{t-1}^d - d_3 r_{t-1}^f) \\ &\quad + \varepsilon_t^f \end{aligned} \tag{2b}$$

$$\begin{aligned}\Delta r_t^d &= c_0'' + c_1''\Delta s_t + c_2''\Delta f_t + c_3''\Delta r_{t-1}^d + c_4''\Delta r_t^f \\ &\quad + c_5''(s_{t-1} - d_1 f_{t-1} - d_2 r_{t-1}^d - d_3 r_{t-1}^f) \\ &\quad + \varepsilon_t^{r^d} \end{aligned} \tag{2c}$$

$$\begin{aligned}\Delta r_t^f &= c_0''' + c_1'''\Delta s_t + c_2'''\Delta f_t + c_3'''\Delta r_t^d + c_4'''\Delta r_{t-1}^f \\ &\quad + c_5'''(s_{t-1} - d_1 f_{t-1} - d_2 r_{t-1}^d - d_3 r_{t-1}^f) \\ &\quad + \varepsilon_t^{r^f} \end{aligned} \tag{2d}$$

where $s_{t-1} - d_1 f_{t-1} - d_2 r_{t-1}^d - d_3 r_{t-1}^f$ is the one-period lagged error-correction term among the four I(1) variables, and $(c_5, c_5', c_5'', c_5''')$ is the vector of adjustment coefficients corresponding to $(\Delta s_t, \Delta f_t, \Delta r_t^d, \Delta r_t^f)$.

The Unbiased Expectations Hypothesis in equation (1) consists of four equations, corresponding to the spot and futures prices, and the domestic and foreign interest rates. Technically, equations (1c)-(1d) are not necessary to specify the Unbiased Expectations Hypothesis, but they are essential for purposes of testing (1a)-(1d) as a special case of (2a)-(2d). The Cost-of-Carry model given in equation (2) also corresponds to the same four variables, albeit in a more general form. In order for equation (2) to reduce to (1), it is necessary that the spot and futures prices be eliminated from the two interest rate equations, (2c)-(2d), and that the two interest rates be eliminated from the spot and futures price equations, (2a)-(2b). Such elimination can arise by deleting the appropriate variables from the short-run components of the models and from all or part of the error-correction term. Thus, the Unbiased Expectations Hypothesis is nested within the Cost-of-Carry model according to the following null hypothesis:

$$\begin{aligned} H_0 : c_3 &= c_4 = d_2 = d_3 = c_3' = c_4' = c_1'' \\ &= c_2'' = c_5'' = c_1''' = c_2''' = c_5''' = 0. \end{aligned} \tag{3}$$

Notice that, under H_0, r_{t-1}^d and r_{t-1}^f are deleted from the error-correction term in (2a)- (2b), whereas the error-correction term is deleted altogether from (2c)-(2d). If the errors in (2) are jointly normally distributed, the null hypothesis can be tested using a likelihood-based test, which is distributed as $\chi^2(12)$ under H_0. The Lagrange Multiplier test of normality, LM(N), will be used to determine if the marginal distributions of the errors of each of the equations is normally distributed.

An extension to the analysis given above involves the case in which there are two cointegrating vectors. Economic rationale would suggest a cointegrating relation between the futures and spot prices, and also between the domestic and foreign interest rates. When two such cointegrating relationships are present among the four I(1) variables, the Unbiased Expectations Hypothesis is given as equations (1a')-(1d') in the following system:

$$\begin{aligned}\Delta s_t &= a_0 + a_1\Delta s_{t-1} + a_2\Delta f_t \\ &\quad + a_3(s_{t-1} - b_1 f_{t-1}) + \varepsilon_t^s \end{aligned} \tag{1a'}$$

$$\begin{aligned}\Delta f_t &= a_0' + a_1'\Delta s_t + a_2'\Delta f_{t-1} \\ &\quad + a_3'(s_{t-1} - b_1 f_{t-1}) + \varepsilon_t^f \end{aligned} \tag{1b'}$$

$$\begin{aligned}\Delta r_t^d &= a_0'' + a_1''\Delta r_{t-1}^d + a_2''\Delta r_t^f \\ &\quad + a_3''(r_{t-1}^d - b_1' r_{t-1}^f) + \varepsilon_t^{r^d} \end{aligned} \tag{1c'}$$

$$\begin{aligned}\Delta r_t^f &= a_0''' + a_1'''\Delta r_t^d + a_2'''\Delta r_{t-1}^f \\ &\quad + a_3'''(r_{t-1}^d - b' r_{t-1}^f) + \varepsilon_t^{r^f} \end{aligned} \tag{1d'}$$

where $s_{t-1} - b_1 f_{t-1}$ is the one-period lagged error-correction term between the spot and futures prices, $r_{t-1}^d - b' r_{t-1}^f$ is the one-period lagged error-correction term between the domestic and foreign interest rates, and $(a_3, a_3', a_3'', a_3''')$ is the vector of adjustment coefficients corresponding to $(\Delta s_t, \Delta f_t, \Delta r_t^d, \Delta r_t^f)$. The time series specifications of (1c')-(1d'), which include the cointegrating relationship between the two interest rate variables, is arbitrary, but nevertheless serves the purpose of enabling the Unbiased Expectations Hypothesis given by equations (1a')-(1d') to be tested against a Cost-of-Carry model with two cointegrating vectors. Indeed, the Unbiased Expectations Hypothesis is consistent with any relationship between the two interest rates. For example, if interest rate parity holds between the domestic and foreign countries, then equations (1c)-(1d) will be misspecified, but the Unbiased Expectations Hypothesis in (1a)-(1b) could still hold.

The Cost-of-Carry model formulated with two cointegrating vectors is given as follows:

$$\begin{aligned}\Delta s_t = g_0 + g_1\Delta s_{t-1} + g_2\Delta f_t + g_3\Delta r_t^d \\ + g_4\Delta r_t^f + g_5(s_{t-1} - d_1' f_{t-1}) \\ + g_6(r_{t-1}^d - d_2' r_{t-1}^f) + \varepsilon_t^s\end{aligned} \qquad \text{(2a')}$$

$$\begin{aligned}\Delta f_t = g_0' + g_1'\Delta s_t + g_2'\Delta f_{t-1} + g_3'\Delta r_t^d \\ + g_4'\Delta r_t^f + g_5'(s_{t-1} - d_1' f_{t-1}) \\ + g_6'(r_{t-1}^d - d_2' r_{t-1}^f) + \varepsilon_t^f\end{aligned} \qquad \text{(2b')}$$

$$\begin{aligned}\Delta r_t^d = g_0'' + g_1''\Delta s_t + g_2''\Delta f_t + g_3''\Delta r_{t-1}^d \\ + g_4''\Delta r_t^f + g_5''(s_{t-1} - d_1' f_{t-1}) \\ + g_6''(r_{t-1}^d - d_2' r_{t-1}^f) + \varepsilon_t^{r^d}\end{aligned} \qquad \text{(2c')}$$

$$\begin{aligned}\Delta r_t^f = g_0''' + g_1'''\Delta s_t + g_2'''\Delta f_t + g_3'''\Delta r_t^d \\ + g_4'''\Delta r_{t-1}^f + g_5'''(s_{t-1} - d_1' f_{t-1}) \\ + g_6'''(r_{t-1}^d - d_2' r_{t-1}^f) + \varepsilon_t^{r^f}\end{aligned} \qquad \text{(2d')}$$

Where $(g_5, g_5', g_5'', g_5''')$ is the vector of adjustment coefficients of the cointegrating vector between the spot and futures prices corresponding to $(\Delta s_t, \Delta f_t, \Delta r_t^d, \Delta r_t^f)$, and $(g_6, g_6', g_6'', g_6''')$ is the vector of adjustment coefficients of the cointegrating vector between the two interest rates corresponding to $(\Delta s_t, \Delta f_t, \Delta r_t^d, \Delta r_t^f)$.

In the case of two cointegrating vectors, the Unbiased Expectations Hypothesis in equation (1') consists of four equations, corresponding to the spot and futures prices, and the domestic and foreign interest rates. Equations (1c')-(1d') are not necessary to specify the Unbiased Expectations Hypothesis, but they are essential for purposes of testing (1a')-(1d') as a special case of (2a')-(2d'). In order for equation (2') to reduce to (1'), it is necessary that the spot and futures prices, and the error-correction term between the spot and futures prices, be eliminated from the two interest rate equations, (2c')-(2d'), and that the two interest rates, and the error-correction term between the two interest rates, be eliminated from the spot and futures price equations, (2a')-(2b'). Such elimination can arise by deleting the appropriate variables from the short-run components of the models and the error-correction terms. Thus, in the case of two cointegrating vectors, the Unbiased Expectations Hypothesis is nested within the Cost-of-Carry model according to the following null hypothesis:

$$\begin{aligned}H_0 : g_3 = g_4 = g_6 = g_3' = g_4' = g_6' = g_1'' \\ = g_2'' = g_5'' = g_1''' = g_2''' = g_5''' = 0.\end{aligned} \qquad \text{(3')}$$

Notice that, under H_0, the error-correction term between the two interest rates is deleted from (2a')-(2b'), and the error-correction term between the spot and futures prices is deleted from (2c')-(2d'). If the errors in (2') are jointly normally distributed, the null hypothesis can be tested using a likelihood-based test, which is distributed as $\chi^2(12)$ under H_0. As in the earlier case with one cointegrating vector, the Lagrange Multiplier test of normality, LM(N), will be used to determine if the marginal distributions of the errors of each of the equations is normally distributed.

3. DATA

The futures contracts used in this study are the Australian dollar futures contracts traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange. Data on the futures and spot prices of the Australian dollar are in natural logarithms, while data on the US 90-day Treasury spot and Australian 90-day bank accepted bill rates are in levels. The foreign risk-free rate of interest in the Cost-of-Carry model is represented by the 90-day bank accepted bill rates, with the US Treasury bill rate being used as the domestic risk-free rate of interest.

Due to the nature of futures contracts, prices obtained on futures contracts reflect a "stale price effect" when a single contract is analyzed. This effect is the occurrence of a dramatic fall in the open interest and trading activity as the maturity date of the particular contract is reached. Prices of futures contracts in the last days prior to maturity as said to be stale. To overcome this effect, the analysis of futures prices is typically performed using several contracts over a longer time span. This approach will, however, result in overlapping contracts since, on any trading day, several contracts with different maturities may be traded simultaneously.

The issue as to the handling of overlapping contracts in futures data, coupled with the stale price effect, remains unresolved. Different approaches have been proposed and used in various studies such as Clark (1973) and Hakkio (1981). In this paper, the futures price data cover a total of twenty three contracts between 6 September 1990 and 17 July 1996. Continuous time series of futures prices are obtained by rolling over the current futures contract two weeks before maturity. Contracts are linked by excluding the last two weeks prior to delivery of the current contract, using volume as a guide. Following this procedure, a total of 1421 observations on the Australian dollar currency futures series are obtained.

An examination of the time plots of futures and spot prices reveals a structural break in the futures price series at observation 747 in the sample period. Consequently, the empirical analysis is based on the following three sample sets:

Set 1: the full sample with one structural break;

Set 2: the full sample ignoring the structural break;

Set 3: the following two separate sub-samples:
Set 3A: observations 1 to 747;
Set 3B: observations 748 to 1421.

The graphs for the levels of the four variables are given in Figures 1 to 4, and their first differences are given in Figures 5 to 8.

4. EMPIRICAL RESULTS

Data on the futures and spot price series exhibit a significant structural break for the full sample at observation 747, corresponding to 30 September 1993, which is around the period when recovery of the Australian economy was being consolidated in late 1993. As the result of a structural break, tests of non-stationarity are affected in that the Dickey-Fuller and Phillips-Perron test statistics are biased towards the non-rejection of a unit root. In testing the stationarity of the four variables, the unit root testing strategy proposed by Perron (1989), which tests for unit roots in the presence of structural breaks, is applied.

The structural break point at observation 747 is applied to all four variables. Two different types of unit root tests are applied to the full sample, with and without accommodating the structural break, and the two separate sub-samples. Perron's unit root testing strategy is employed for sample set 1 to determine the order of integration of all four variables in the presence of a single structural break. Augmented Dickey-Fuller (ADF) tests are used for sample sets 2 and 3, when the analysis involves either the full sample period without the structural break, as in set 2, or when the analysis is conducted as two separate sub-samples, as in set 3.

For sample set 1, the unit root testing strategy is conducted by regressing each of the four variables separately according to model (B) in Perron (1989). This model is estimated as :

$$y_t = \hat{\mu}^B + \hat{\theta}^B DU_t + \hat{\beta}^B t + \hat{\gamma}^B DT_t^* + \hat{\alpha}^B y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + \hat{e}_t \qquad (4)$$

In equation (4), the ADF regression specifies a null hypothesis of a permanent change in the magnitude of the drift term versus the alternative of a change in the slope of the trend, t; $DU = 1$ if $t > \tau$ and is zero otherwise; $DT_t^* = t - \tau$ for $t > \tau$ and is zero otherwise; τ refers to the time of the break, that is, the period in which the change in the parameters of the trend function occurs; $\hat{\mu}^B$ is the estimated drift term in the regression; and Δy_{t-i} are the lagged first differences to account for any residual serial correlation. The pth-order ADF statistic for testing $\alpha^B = 1$, denoted ADF(p), is given by the t-ratio of $(\hat{\alpha}^B - 1)$ in equation (4).

To determine k, the order of the regression, an initial lag length of ten is used in the ADF regression, given by equation (4), and the tenth lag is tested for significance using the asymptotic t-ratio. If the tenth lag is found to be insignificant, it is omitted from the regression, and the ninth lag is tested for significance. Applying this procedure until a significant lag length is determined, the lag length to be used in the ADF test is readily obtained.

Results of the ADF test for all four variables and their first differences are presented in Tables 1 and 2. A Newey-West covariance matrix, denoted NW, is used for one of the ADF regressions, namely for Australian rates in sample set 1, when LM tests for the presence of heteroskedasticity and serial correlation are found to be significant. Dickey-Fuller (DF) or ADF statistics obtained from the test are compared with critical values at the 5% significance level for the appropriate value of λ, the ratio of the pre-break sample size to the total sample size.

For all four variables, the unit root null hypothesis is not rejected at the 5% level in sample set 1. The results suggest that all four variables contain a unit root and are defined by a stochastic shift occurring at the break point, denoted by DL.

In sample sets 2 and 3, the pth-order ADF statistic, denoted ADF(p), is given by the t-ratio of the OLS estimate of β in the ADF regression (5) below (see Campbell and Perron (1991)):

$$\Delta y_t = \alpha + \gamma t + \beta y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + u_t \qquad (5)$$

where Δy_t is the first difference of the variable, α is the constant of the regression, γ is the coefficient of the deterministic trend, Δy_{t-i} denotes the lagged first differences, δ_i is the coefficient of the lagged first differences, and u_t is the error term. The time trend is omitted when the ADF statistics, with and without a trend, are not substantially different from one another. To determine the order of the ADF equation, the standard procedure for testing the significance of the lagged first differences is adopted. The ADF statistics obtained are compared with the simulated critical values given in MacKinnon (1991).

For all four variables, the null hypothesis of a unit root is not rejected for sample sets 2, 3A and 3B. A deterministic trend is present for the Australian rate of interest in sample set 2, for all four variables in sample set 3A, and for all variables other than the spot price in sample set 3B. When unit root tests are conducted for the first differences of the four variables, the null hypothesis of a unit root is rejected for sample sets 2, 3A and 3B. Since there did not appear to be any structural break in the data on the basis of Figures 5-8, no unit root tests were conducted for sample set 1.

Estimates of the error-correction terms for the Unbiased Expectations Hypothesis and the Cost-of-Carry model are given in Tables 3A and 3B, respectively. In Table 3A, the coefficient of the lagged futures price in the cointegrating relation is very close to unity, lying in the range (-1.0214, -1.0092). This suggests that the spot and futures prices are cointegrated, with the cointegrating vector closely approximated by (1,-1). Brenner and Kroner (1995) document recent empirical studies of currency futures markets that show the cointegrating vector is very close to (1, -1), with the coefficient of the lagged futures price in the cointegrating relation lying in the range (-1.03, -0.95). The values of the cointegrating vector obtained in this paper are, therefore, consistent with recent empirical results.

This empirical result is further supported by the results in Table 3B, which show that the coefficient of the lagged futures price in the cointegrating relation is also very close to unity. Thus, the spot and futures prices would seem to be cointegrated, with their respective cointegrating coefficients given approximately by 1 and –1. However, with the presence of the lagged domestic and foreign interest rates in the cointegrating relation, where the cointegrating coefficients are highly significant at the 1% level, as well as being similar in estimated magnitude but of opposite signs, the Cost-of-Carry model would seem to be preferred to the Unbiased Expectations Hypothesis.

Formal tests of the two systems against each other are given for the four sample sets in Table 4A. The calculated Wald test statistics, which are asymptotically distributed as $\chi^2(12)$ under the null hypothesis given in equation (3), are each highly significant. Such strong rejections of the zero restrictions on the Cost-of-Carry model suggest clearly that the Cost-of-Carry model dominates the system underlying the Unbiased Expectations Hypothesis for the four sample sets considered.

Similar results are obtained in the case of two cointegrating vectors. Table 4B presents the Wald test statistics of zero restrictions on the Cost-of-Carry model, given in equation (3'). These results support the Cost-of-Carry model, which suggests that the Cost-of-Carry model also dominates the Unbiased Expectations Hypothesis in the case where there are two cointegrating vectors.

5. CONCLUSION

Two standard models of futures pricing, namely the Unbiased Expectations Hypothesis and the Cost-of-Carry model, have been tested for the pricing of Australian dollar futures contracts traded on the International Monetary Market of the Chicago Mercantile Exchange. Cointegrating relationships between the Australian dollar spot and futures prices, as well as the US and Australian risk-free rates of interest, suggest an error-correction representation for the Cost-of-Carry model which, with appropriate zero restrictions, yields the error-correction formulation for the Unbiased Expectations Hypothesis.

A structural break in the futures price series permits testing of appropriate models for the full sample in the presence of the break, for the full sample without explicitly modelling the break, and for two separate sub-samples created by the structural break. The restricted and unrestricted Cost-of-Carry formulations are estimated for all sample sets,.and on the basis of the tests of zero restrictions, the Cost-of-Carry model is found to be empirically superior to the Unbiased Expectations Hypothesis for the four sample sets considered, regardless of the number of cointegrating relations.

ACKNOWLEDGEMENTS

The first author gratefully acknowledges the Australian Department of Employment, Education, Training and Youth Affairs for an Overseas Postgraduate Research Award, the C.A. Vargovic Memorial Fund at UWA, and the Faculties of Economics and Commerce, Education and Law at UWA for an Individual Research Grant; the second author wishes to acknowledge the financial support of the Australian Research Council; and the third author would like to acknowledge partial financial support from the Chinese University of Hong Kong and UWA.

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Table 1: Unit root tests of levels

Sample set	Test	Spot	Futures	US rates	Aust rates
1	Trend?	No	No	No	No
	ADF lag length	3	3	9	1
	Covariance formula	OLS	OLS	OLS	NW
	ADF statistic	-3.59	-3.71	-1.22	-1.26
	Critical value	-3.96	-3.96	-3.96	-3.96
2	Trend?	No	No	No	Yes
	ADF lag length	3	3	9	1
	ADF statistic	-2.08	-2.06	-2.40	-2.32
	Critical value	-2.86	-2.86	-2.86	-3.42
3A	Trend?	Yes	Yes	Yes	Yes
	ADF lag length	3	0	1	0
	ADF statistic	-2.31	-2.63	-1.08	-1.19
	Critical value	-3.42	-3.42	-3.42	-3.42
3B	Trend?	No	Yes	Yes	Yes
	ADF lag length	0	5	6	5
	ADF statistic	-2.53	-2.73	-0.67	-0.49
	Critical value	-2.87	-3.42	-3.42	-3.42

Note: Trend denotes the deterministic trend; NW denotes the Newey-West covariance matrix formula. Unless otherwise specified, the OLS covariance formula is used in the calculation of the test statistics.

Table 2: Unit root tests of first differences of variables

Sample set	Test	Spot	Futures	US rates	Aust rates
2	ADF lag length	2	2	4	2
	ADF statistic	-24.9	-24.9	-17.7	-21.4
	Critical value	-2.86	-2.86	-2.86	-2.86
3A	ADF lag length	2	2	0	0
	ADF statistic	-17.3	-17.3	-29.1	-25.6
	Critical value	-2.87	-2.87	-2.87	-2.87
3B	ADF lag length	4	2	5	8
	ADF statistic	-13.5	-16.7	-11.8	-7.7
	Critical value	-2.87	-2.87	-2.87	-2.87

Note: The ADF tests are conducted without a time trend since the t-statistics with and without a trend are not substantially different.

Table 3A: Estimates of the error-correction term in the Unbiased Expectations Hypothesis System, (1a)-(1b), for all sample sets

Estimate	1	2	3A	3B
b_1	1.0204	1.0214	1.0260	1.0092

Note: The values in the tables are the estimates of b_1 in the error-correction term given by $s_{t-1} - b_1 f_{t-1}$. All estimates are highly significant at the 1% level.

Table 3B: Estimates of the error-correction term in the Cost-of-Carry System, (2a)-(2d), for all sample sets

Estimate	1	2	3A	3B
d_1	1.0063	1.0045	1.0122	0.9811
d_2	-0.0074	-0.0063	-0.0106	-0.0063
d_3	0.0048	0.0049	0.0062	0.0055

Note: The values in the tables are the estimates of d_1, d_2 and d_3 in the error-correction term given by $s_{t-1} - d_1 f_{t-1} - d_2 r^d_{t-1} - d_3 r^f_{t-1}$. All estimates are highly significant at the 1% level.

Table 4A: Wald tests of the null hypothesis in equation (3) for all sample sets

Wald Statistic	1	2	3A	3B
$\chi^2(12)$	284.6	496.6	138.7	189.7

Note: The values in the table are the calculated Wald test statistics, which are asymptotically distributed as $\chi^2(12)$ under the null hypothesis given in equation (3). All calculated statistics are highly significant at the 1% level.

Table 4B: Wald tests of the null hypothesis in equation (3') for all sample sets

Wald Statistic	1	2	3A	3B
$\chi^2(12)$	72.8	87.9	44.0	45.8

Note: The values in the table are the calculated Wald test statistics, which are asymptotically distributed as $\chi^2(12)$ under the null hypothesis given in equation (3'). All calculated statistics are highly significant at the 1% level.